# INCLUSIVE SEMILEPTONIC DECAYS AND THE STRUCTURE OF B MESONS

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## Abstract

A field theoretic description for inclusive semileptonic B meson decays is formulated. We argue that large regions of the phase spaces for the decays are dominated by distances near the light cone. The light-cone dominance allows to incorporate nonperturbative QCD effects in a distribution function. A one-to-one correspondence with the heavy quark effective theory is developed, which can estimate the first two moments of the distribution function. These conditions are useful but not restrictive enough to specify the distribution function, which must still be determined from experiment. Several model-independent predictions, such as scaling, sum rules of the hadronic structure functions and relations among them, are made. General formulas for the differential decay rates on several variables are presented, which are used for calculating the electron energy spectra with an Ansatz for the light-cone distribution function.

# 1 Introduction

The study of semileptonic and inclusive B meson decays is interesting on several accounts. They are some of the simplest decays to study theoretically and experimentally. For semileptonic processes the structure of the lepton current is completely known and the inclusive hadronic tensor involves a sum over all final states, which makes the decay products incoherent. Inclusive semileptonic B meson decays are useful for obtaining parameters of the electroweak theory. Two standard model parameters $V_{cb}$ and $V_{ub}$ are extracted from them. In fact, the charmless inclusive semileptonic decay of B mesons is the main experimental source at present which determines $V_{ub}$. This follows from a measurement of the endpoint spectrum of the charged lepton energy, whose better understanding is highly desirable.

The hadronic tensor for the decays involves short and long distance contributions, where confinement effects are also important. There are, however, several features of the decay which make its theoretical analysis possible. The mass of the B-meson is large enough in comparison to 1 GeV, so that the mass of the virtual W boson is large enough to produce a commutator of two weak currents which are at light-cone distances relative to each other. This occurs for large but not all regions of phase space, which allows us to use the methods of deep inelastic scattering (DIS) and replace the commutator of the two currents with its singularity on the light cone times an operator bilocal in the quark fields. The matrix element of the bilocal operator between B-meson states contains nonperturbative QCD corrections. Its light-cone Fourier transformation is related to a distribution function, in direct analogy to DIS. The distribution function was discussed earlier [1, 2, 3] and since it involves a heavy quark it is expected to be large in the region where the b quark carries a large fraction of the B-mesons momentum.

The hadronic matrix element has another feature which was discussed recently. Namely, it contains a heavy quark and several properties can be established in the heavy quark effective theory (HQET) [4]. In this paper we adopt the HQET and show that there is a correspondence between results obtained by other groups [5, 6, 7, 8, 9] based on the HQET

and moments of the distribution function defined on the light cone. The first two moments of the distribution function can be estimated, which determine the mean value and the variance of it. These two properties are useful but not restrictive enough to determine the distribution function completely. We find that in order to reproduce the decay spectrum, we need more detailed knowledge of the distribution function. For this reason we find it appropriate to introduce the distribution function over the whole range of its variable, instead of its first few moments, and try to determine it from experiment.

In this paper we present a field theoretic prescription for inclusive semileptonic B meson decays and determine properties of the distribution function. We intend to justify, in terms of field theory, the parton model for inclusive B decays [1, 2, 3] and to identify systematic procedures for improving upon the parton model predictions. The advantage of this approach is that model-independent aspects of the analysis can be clearly separated from model-dependent ones. Several model-independent predictions are obtained. Among them is scaling: away from the boundary of the phase space, the hadronic structure functions depend on the scaling variable

$$\xi_+ = [q \cdot P_B + \sqrt{(q \cdot P_B)^2 - M_B^2(q^2 - m_q^2)}]/M_B^2$$

only, and not on the momentum transfer squared $q^2$ directly.

The paper is planed as follows. In sect. 2 we give the kinematics and the general formalism for inclusive semileptonic B meson decays. In sect. 3 we argue that very large domains of phase space are dominated by light-cone distances of the two weak currents. The distribution function is introduced in sect. 4, while in sect. 5 the general properties and the physical implication of the distribution function are discussed. In sect. 6 we use the techniques of the operator product expansion (OPE) and the HQET to estimate moments of the light-cone distribution function. Sections 7 and 8 include predictions. Some model-independent predictions are presented in sect. 7. We give the triple differential decay rates for both $\bar{B} \to e\bar{\nu}_e X_u$ and $\bar{B} \to e\bar{\nu}_e X_c$ channels in sect. 8. In order to produce quantitative features of the data, we propose an Ansatz for the light-cone distribution function consistent with its known properties. We evaluate the electron energy spectra using the Ansatz. The

conclusions are in sect. 9.

## 2   Kinematics

We consider the inclusive semileptonic decays $\bar{B} \to l\bar{\nu}_l X_q$, where $l = e, \mu$, or $\tau$ and $X_q$ is any possible hadronic final state containing a charm quark $(q = c)$ or an up quark $(q = u)$. The decays are produced by weak interactions. The partial decay width is given by

$$d\Gamma = \frac{1}{2E_B} \sum_n |\mathcal{M}|^2 \frac{d^3 P_l}{(2\pi)^3 2E_l} \frac{d^3 P_\nu}{(2\pi)^3 2E_\nu} \Big[ \prod_{i=1}^{n} \frac{d^3 P_i}{(2\pi)^3 2E_i} \Big] (2\pi)^4 \delta^4(P_B - q - \sum_{i=1}^{n} P_i), \quad (1)$$

where $q$ is the four-momentum of the virtual W boson, $P$ and $E$ denote the four-momentum and energy, respectively. Their subscripts $B$, $l$, $\nu$, and $i$ correspond to the B meson, the charged lepton, the neutrino, and the final state particle, respectively. The summation on $n$ in Eq.(1) and hereafter implies a sum over all possible final hadronic states $|n\rangle$. At the tree level, the decay amplitude is given by

$$\mathcal{M} = V_{qb} \frac{G_F}{\sqrt{2}} \bar{u}(P_l)\gamma^\mu(1 - \gamma_5)v(P_\nu)\langle n \left| j_\mu(0) \right| B\rangle, \quad (2)$$

where $V_{qb}$ are the matrix elements of the CKM matrix and the relevant charged weak current is

$$j_\mu(x) =: \bar{q}(x)\gamma_\mu(1 - \gamma_5)b(x) :. \quad (3)$$

$|B\rangle$ is the B-meson state with momentum $P_B^\mu$ and is normalized according to $\langle B | B \rangle = 2E_B(2\pi)^3\delta^3(\mathbf{0})$. Perturbative QCD corrections, which were studied in [10, 11, 12] and more recently in [13, 14], are not included. They can be added in an analysis of the data as a perturbation. For unpolarized leptons the partial decay width can be written as

$$d\Gamma = \frac{G_F^2 |V_{qb}|^2}{(2\pi)^5 E_B} L^{\mu\nu} W_{\mu\nu} \frac{d^3 P_l}{2E_l} \frac{d^3 P_\nu}{2E_\nu}, \quad (4)$$

where $L_{\mu\nu}$ is the leptonic tensor

$$L^{\mu\nu} = 2(P_l^\mu P_\nu^\nu + P_\nu^\mu P_l^\nu - g^{\mu\nu} P_l \cdot P_\nu + i\varepsilon^{\mu\nu}{}_{\alpha\beta} P_l^\alpha P_\nu^\beta). \quad (5)$$

4

$W_{\mu\nu}$ is the hadronic tensor

$$W_{\mu\nu} = \sum_n \int \Big[ \prod_{i=1}^n \frac{d^3 P_i}{(2\pi)^3 2E_i} \Big] (2\pi)^3 \delta^4 \Big( P_B - q - \sum_{i=1}^n P_i \Big) \langle B \big| j_\nu^\dagger(0) \big| n \rangle \langle n \left| j_\mu(0) \right| B \rangle. \quad (6)$$

It is useful to express the hadronic tensor in terms of a current commutator

$$W_{\mu\nu} = -\frac{1}{2\pi} \int d^4 y\, e^{iq\cdot y} \langle B \big| [j_\mu(y), j_\nu^\dagger(0)] \big| B \rangle. \quad (7)$$

Thus the commutator of two weak currents is relevant in inclusive semileptonic B meson decays. Furthermore, we shall see that the commutator of currents $[j_\mu(y), j_\nu^\dagger(0)]$ near the light-cone $y^2 = 0$ plays a central role in inclusive semileptonic B meson decays.

Generally, the hadronic tensor can be decomposed by introducing the hadronic structure functions $W_i(q^2, q \cdot P_B)$ with two scalar variables chosen to be $q^2$ and $q \cdot P_B$,

$$
\begin{aligned}
W_{\mu\nu}(P_B, q) =\ & -g_{\mu\nu} W_1(q^2, q \cdot P_B) + \frac{P_{B\mu} P_{B\nu}}{M_B^2} W_2(q^2, q \cdot P_B) \\
& -i\varepsilon_{\mu\nu\alpha\beta} \frac{P_B^\alpha q^\beta}{M_B^2} W_3(q^2, q \cdot P_B) + \frac{q_\mu q_\nu}{M_B^2} W_4(q^2, q \cdot P_B) \\
& + \frac{P_{B\mu} q_\nu + q_\mu P_{B\nu}}{M_B^2} W_5(q^2, q \cdot P_B),
\end{aligned}
\quad (8)
$$

where $M_B$ is the B-meson mass. The hadronic structure functions $W_i(q^2, q \cdot P_B)$ characterize the structure of the decaying B meson. Nonperturbative QCD effects for the inclusive process under consideration are incorporated in them.

Finally, the triple differential decay rate is obtained from the kinematical analysis

$$\frac{d^3\Gamma}{dE_l dq^2 dq_0} = \frac{G_F^2 |V_{qb}|^2}{32\pi^3 E_B} L^{\mu\nu} W_{\mu\nu}, \quad (9)$$

where the contraction of the hadronic with the leptonic tensor yields

$$
\begin{aligned}
L^{\mu\nu} W_{\mu\nu} =\ & 2(q^2 - M_l^2) W_1(q^2, q \cdot P_B) \\
& + [4P_l \cdot P_B\, q \cdot P_B - 4(P_l \cdot P_B)^2 - M_B^2 q^2 + M_B^2 M_l^2] W_2(q^2, q \cdot P_B)/M_B^2 \\
& + 2[(q^2 + M_l^2) q \cdot P_B - 2q^2 P_l \cdot P_B] W_3(q^2, q \cdot P_B)/M_B^2 \\
& + M_l^2(q^2 - M_l^2) W_4(q^2, q \cdot P_B)/M_B^2 \\
& + 4M_l^2(q \cdot P_B - P_l \cdot P_B) W_5(q^2, q \cdot P_B)/M_B^2,
\end{aligned}
\quad (10)
$$

where $M_l$ denotes the charged lepton mass. There are three independent kinematical variables in this inclusive phenomenology, for which we choose $P_l \cdot P_B$, $q \cdot P_B$, and $q^2$. We see that for the massless lepton case only three hadronic structure functions $W_1(q^2, q \cdot P_B)$, $W_2(q^2, q \cdot P_B)$, and $W_3(q^2, q \cdot P_B)$ contribute. We will proceed to investigate the hadronic structure functions in a way as suggested by the light-cone dominance.

## 3 Light-Cone Dominance

B mesons are heavy. This implies that its decay dynamics is analogous to that of deep inelastic scattering, that is, the light-cone dynamics dictates the inclusive semileptonic B meson decay. To see this we start with an analysis of the hadronic tensor.

According to the causality requirement the commutator in Eq.(7) has to vanish for space-like $y$, i.e., $[j_\mu(y), j_\nu^\dagger(0)] = 0$, for $y^2 < 0$, and hence the integrand in Eq.(7) has a support only for $y^2 \geq 0$.

Taking $q = (q_0, 0, 0, q_3)$, we have

$$q \cdot y = q_0 y_0 - q_3 y_3 = \frac{1}{2}(q_0 + q_3)(y_0 - y_3) + \frac{1}{2}(q_0 - q_3)(y_0 + y_3). \tag{11}$$

The dominant contribution to the Fourier transform of the commutator in Eq.(7) comes from domains with less rapid oscillations, i.e. $q \cdot y = \mathcal{O}(1)$; hence

$$y_0 - y_3 \sim \frac{1}{q_0 + q_3}, \tag{12}$$

$$y_0 + y_3 \sim \frac{1}{q_0 - q_3}, \tag{13}$$

and

$$y^2 = y_0^2 - y_1^2 - y_2^2 - y_3^2 \leq y_0^2 - y_3^2 \sim \frac{1}{q_0^2 - q_3^2} = \frac{1}{q^2}. \tag{14}$$

Therefore, the dominant contribution to the integral (7) results from the range $0 \leq y^2 \leq 1/q^2$. This implies that as long as $q^2$ is large enough, $q^2 \geq q_0^2$, the decays take place near the light-cone $y^2 = 0$, where $q_0^2$ is a reference scale and experience shows that $q_0^2 \simeq 1\ GeV^2$. The scale $q_0^2$ should be determined ultimately by experiment.

6

For inclusive semileptonic B meson decays $q^2$ varies in the physical range of

$$M_l^2 \leq q^2 \leq (M_B - M_{X_{min}})^2, \tag{15}$$

where $M_{X_{min}}$ is the minimum value of the final hadronic invariant mass. The light-cone domain $q_0^2 \leq q^2 \leq (M_B - M_{X_{min}})^2$ covers most of the phase space, since the B meson is so heavy that the interval $(M_B - M_{X_{min}})^2 - q_0^2 \gg q_0^2 - M_l^2$. Subsequently, in inclusive semileptonic B meson decays the light-cone contribution dominates over all other nonperturbative QCD contributions. Contributions far from the light cone are suppressed dynamically and kinematically. The leading approximation of nonperturbative QCD effects should be more reliable for the charmless decays $\bar{B} \to l\bar{\nu}_l X_u$, where $M_{X_{min}}$ is negligible, and/or for the decays to the final states containing a $\tau$ lepton with the mass $M_\tau = 1.777 \ GeV$. For both cases nonperturbative QCD contributions far from the light cone are more seriously suppressed kinematically.

# 4    Distribution Functions

Applying Wick's theorem one obtains

$$
\begin{aligned}
[j_\mu(y), j_\nu^\dagger(x)] &= \bar{q}(y)\gamma_\mu(1 - \gamma_5)\{b(y), \bar{b}(x)\}\gamma_\nu(1 - \gamma_5)q(x) \\
&\quad -\bar{b}(x)\gamma_\nu(1 - \gamma_5)\{q(x), \bar{q}(y)\}\gamma_\mu(1 - \gamma_5)b(y).
\end{aligned} \tag{16}
$$

The quark pairs $q\bar{q}$ could be either charm or up quarks. In the case of charm quarks the matrix element between B mesons is very small, because there is no spectator charm quarks in $B^-(b\bar{u})$ and $\bar{B}^0(b\bar{d})$ mesons. The up-quark bilocal operator has a matrix element only between $B^-$ states, but this term is suppressed for the reason given after Eq.(23). For the matrix element we keep only the second term in Eq.(16). The hadronic matrix element of the current commutator becomes

$$\langle B \left| [j_\mu(y), j_\nu^\dagger(0)] \right| B \rangle = -\langle B \left| \bar{b}(0)\gamma_\nu(1 - \gamma_5)\{q(0), \bar{q}(y)\}\gamma_\mu(1 - \gamma_5)b(y) \right| B \rangle. \tag{17}$$

After some calculation, the expression (17) is transformed into

$$\langle B \left| [j_\mu(y), j_\nu^\dagger(0)] \right| B \rangle = 2(S_{\mu\alpha\nu\beta} - i\varepsilon_{\mu\alpha\nu\beta})[\partial^\alpha \Delta_q(y)]\langle B \left| \bar{b}(0)\gamma^\beta(1 - \gamma_5)b(y) \right| B \rangle, \tag{18}$$

7

where $S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta}$. $\Delta_q(y)$ is the Pauli-Jordan function for a free $q$-quark,

$$i\Delta_q(y) = \int \frac{d^4p}{(2\pi)^3} e^{-ip\cdot y}\varepsilon(p_0)\delta(p^2 - m_q^2). \qquad (19)$$

In Eq.(18) the matrix element is separated in two factors. The first factor contains the light-cone contribution in the form of the propagator and the long-distance part is included in the reduced matrix element. The decomposition is Lorentz covariant and each factor can be calculated in the Lorentz frame of preference. This is analogous to deep inelastic scattering where the production of two currents at light-like distances is expanded in terms of operators times their Wilson coefficients, which are obtained from perturbative QCD.

In the light-cone limit $y^2 \to 0$, the reduced matrix element in Eq.(18) can be expanded in powers of $y^2$ from the general arguments of Lorentz covariance and translation invariance:

$$\langle B \left| \bar{b}(0)\gamma^\beta(1 - \gamma_5)b(y) \right| B\rangle = 4\pi P_B^\beta \sum_{n=0}^{\infty} (y^2)^n F_n(y \cdot P_B). \qquad (20)$$

We define next the Fourier transform of $F_n(y \cdot P_B)$,

$$\phi_n(\xi) = \int d(y \cdot P_B) e^{i\xi y \cdot P_B} F_n(y \cdot P_B). \qquad (21)$$

Near the light cone only $\phi_0(\xi)$ survives, defined as the Fourier transform of the reduced matrix element at light-like separations

$$f(\xi) \equiv \phi_0(\xi) = \frac{1}{4\pi M_B^2} \int d(y \cdot P_B) e^{i\xi y \cdot P_B} \langle B \left| \bar{b}(0)\slashed{P}_B(1 - \gamma_5)b(y) \right| B\rangle \mid_{y^2=0}. \qquad (22)$$

The physical implication and properties of this distribution function will be discussed in the next section.

Finally, the leading contribution to the hadronic tensor is obtained from Eqs.(7), (18) and (22):

$$W_{\mu\nu} = 4(S_{\mu\alpha\nu\beta} - i\varepsilon_{\mu\alpha\nu\beta}) \int d\xi f(\xi)\varepsilon(\xi P_{B0} - q_0)\delta[(\xi P_B - q)^2 - m_q^2](\xi P_B - q)^\alpha P_B^\beta, \qquad (23)$$

where $m_q$ is the mass of the quark in the final state. Repeating the above steps for the first term in Eq.(16) we obtain a distribution function for the up quark, whose $\xi$-variable is

either negative or very close to one. In the former case, the negative $\xi$ is outside the support of the distribution function and when $\xi$ is very close to one the distribution function for a light-quark is very small, so that this contribution will be neglected.

Comparing Eq.(23) with Eq.(8), we are led to the expressions

$$
\begin{align}
W_1(\xi_+, \xi_-) &= 2[f(\xi_+) + f(\xi_-)], && (24)\\
W_2(\xi_+, \xi_-) &= \frac{8}{\xi_+ - \xi_-}[\xi_+ f(\xi_+) - \xi_- f(\xi_-)], && (25)\\
W_3(\xi_+, \xi_-) &= -\frac{4}{\xi_+ - \xi_-}[f(\xi_+) - f(\xi_-)], && (26)\\
W_4(\xi_+, \xi_-) &= 0, && (27)\\
W_5(\xi_+, \xi_-) &= W_3(\xi_+, \xi_-), && (28)
\end{align}
$$

where the dimensionless variables $\xi_\pm$ are defined as

$$
\xi_\pm = [q \cdot P_B \pm \sqrt{(q \cdot P_B)^2 - M_B^2(q^2 - m_q^2)}]/M_B^2. \tag{29}
$$

Hence, the light-cone dominance ascribes the hadronic structure functions to a single universal light-cone distribution function. The variable $\xi_-$ occurs for the first time in the decays of heavy particles and is a consequence of field theory.

The triple differential decay rate (9) can then be written in terms of the light-cone distribution function

$$
\begin{align}
\frac{d^3\Gamma}{dE_l dq^2 dq_0} &= \frac{G_F^2 |V_{qb}|^2}{4\pi^3 E_B} \frac{1}{\xi_+ - \xi_-}\Bigg\{ f(\xi_+)\Big[(\xi_+ - \xi_-)(q^2 - M_l^2)/2 \\
&\quad + \xi_+[4P_l \cdot P_B q \cdot P_B - 4(P_l \cdot P_B)^2 - M_B^2 q^2 + M_B^2 M_l^2]/M_B^2 \\
&\quad - [(q^2 + 3M_l^2)q \cdot P_B - 2(q^2 + M_l^2)P_l \cdot P_B]/M_B^2\Big] \\
&\quad - (\xi_+ \leftrightarrow \xi_-)\Bigg\}. && (30)
\end{align}
$$

# 5 Properties of the Light-Cone Distribution Function

We discuss here some important properties of the light-cone distribution function. The distribution function is normalized to unity:

$$
\begin{aligned}
\int d\xi f(\xi) &= \frac{1}{4\pi M_B^2} \int d\xi d(y \cdot P_B) e^{i\xi y \cdot P_B} \langle B \left| \bar{b}(0) \not{P}_B (1 - \gamma_5) b(y) \right| B \rangle \big|_{y^2=0} \\
&= \frac{1}{2M_B^2} P_B^\mu \langle B \left| \bar{b}(0) \gamma_\mu (1 - \gamma_5) b(0) \right| B \rangle = 1,
\end{aligned}
\tag{31}
$$

due to the conservation of the b quantum number.

We consider next $f(\xi)$ in the rest frame of the B meson. In this frame,

$$
f(\xi) = \frac{1}{2\pi} \int dy_0 e^{i\xi M_B y_0} \langle B \left| b^\dagger(0) P_L b(y_0) \right| B \rangle,
\tag{32}
$$

where the left-handed projection operator $P_L = (1 - \gamma_5)/2$. We insert a complete set of states between quark fields, translate the $y_0$ dependence out of quark fields. Then we get

$$
f(\xi) = \sum_m \delta(M_B - \xi M_B - p_m^0) \left| \langle m \left| b_L(0) \right| B \rangle \right|^2,
\tag{33}
$$

where $b_L = P_L b$. So we see that $f(\xi)$ obeys positivity. The state $|m\rangle$ is physical and must have $0 \leq p_m^0 \leq M_B$, thus $f(\xi) = 0$, for $\xi \leq 0$ and $\xi \geq 1$. Therefore, the support of the light-cone distribution function reads $0 \leq \xi \leq 1$. These results are valid in any frame due to Lorentz invariance, although they are deduced in the B rest frame. Furthermore, we observe from Eq.(33) that $f(\xi)$ is the probability to find in the B meson a $b$ quark with a momentum $\xi P_B$. This is the familiar probabilistic interpretation of the parton model, except it is written in the B rest frame rather than the infinite momentum frame.

It will be convenient to expand the light-cone distribution function in terms of derivatives of delta functions,

$$
f(\xi) = \sum_{n=0}^\infty \frac{(-1)^n}{n!} M_n(\tilde{\xi}) \delta^{(n)}(\xi - \tilde{\xi}).
\tag{34}
$$

Such an expansion is very singular and any finite number of terms cannot represent the differential decay width. The expansion is convenient for comparisons with operator product expansions which also generate sequences with singular terms. The coefficient $M_n(\tilde{\xi})$

is related to the nth moment about a point $\tilde{\xi}$ of the distribution function as follows

$$M_n(\tilde{\xi}) = \int d\xi (\xi - \tilde{\xi})^n f(\xi). \tag{35}$$

It follows, now, that $M_0(\tilde{\xi}) = 1$, the mean value $\mu$ and the variance $\sigma^2$ of the light-cone distribution function can be expressed by the moments:

$$\mu \equiv M_1(0) = \tilde{\xi} + M_1(\tilde{\xi}), \tag{36}$$

$$\sigma^2 \equiv M_2(\mu) = M_2(\tilde{\xi}) - M_1^2(\tilde{\xi}). \tag{37}$$

To sum up, our results so far are quite general. It is shown that the $b$-quark distribution function inside the B meson introduced in the parton model [1, 2, 3] is related to the light-cone Fourier transformation of the bilocal operator between B-meson states (22). This makes clear the connection with the parton model in inclusive B decays. In the next section we employ the techniques of the operator product expansion and the heavy quark effective theory to estimate moments of the light-cone distribution function.

# 6   Moments of the Light-Cone Distribution Function

To estimate moments of the light-cone distribution function, we must calculate the hadronic matrix element $\langle B \left| \bar{b}(0)\gamma^\beta(1 - \gamma_5)b(y) \right| B \rangle$, which involves long distances and hence brings in nonperturbative effects of QCD. The techniques of the OPE and the HQET provide a possibility to calculate it in a systematic way.

We start with the light-cone OPE. Since the b quark is very heavy within the B meson we can extract the large mass scale

$$b(y) = e^{-im_b v \cdot y} b_v(y), \tag{38}$$

where $m_b$ is the b-quark mass and $v$ is the velocity of the initial B meson, defined by $P_B^\mu = M_B v^\mu$ and the rescaled b-quark field $b_v(y)$ is related to the effective field of the HQET by Eq.(43) below. Upon performing the light-cone OPE the hadronic matrix element

becomes

$$\langle B \left| \bar{b}(0)\gamma^{\beta}(1 - \gamma_5)b(y) \right| B \rangle =$$
$$e^{-im_b v \cdot y} \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} y_{\mu_1} \cdots y_{\mu_n} \langle B \left| \bar{b}_v(0)\gamma^{\beta}(1 - \gamma_5)\mathcal{S}[k^{\mu_1} \cdots k^{\mu_n}]b_v(0) \right| B \rangle \quad (39)$$

with $k_{\mu} = iD_{\mu}$. $\mathcal{S}$ denotes the symmetrization. This OPE keeps the leading twist operators with higher twist effects being neglected. The advantage of this expansion is twofold. First, the Lorentz structure of the matrix element allows us to express it in terms of the B-meson momentum

$$\langle B \left| \bar{b}_v(0)\gamma^{\beta}(1 - \gamma_5)\mathcal{S}[k^{\mu_1} \cdots k^{\mu_n}]b_v(0) \right| B \rangle =$$
$$2(C_{n0}P_B^{\beta}P_B^{\mu_1} \cdots P_B^{\mu_n} + \sum_{i=1}^{n} M_B^2 C_{ni}g^{\beta\mu_i}P_B^{\mu_1} \cdots P_B^{\mu_{i-1}}P_B^{\mu_{i+1}} \cdots P_B^{\mu_n}) +$$
$$terms\ with\ g^{\mu_i\mu_j}. \quad (40)$$

Terms with $g^{\mu_i\mu_j}$ can be omitted on the light cone. Second, we can estimate some terms in the HQET as shown below. Substituting Eqs.(39) and (40) into Eq.(22) we have

$$f(\xi) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} (\sum_{i=0}^{n} C_{ni})\delta^{(n)}(\xi - \frac{m_b}{M_B}). \quad (41)$$

Comparing with Eq.(34), the nth moment about the point $\tilde{\xi} = m_b/M_B$ of the light-cone distribution function is related to the expansion coefficients as following

$$M_n(m_b/M_B) = \sum_{i=0}^{n} C_{ni}. \quad (42)$$

It is straightforward to note that the moment $M_0(m_b/M_B) = C_{00}$ is exactly equal to 1.

We now go to the second step further. We employ the HQET to estimate other expansion coefficients. In this effective theory the rescaled b-quark field $b_v(x)$ is expressed by the velocity-dependent heavy quark field $h_v(x)$ by means of an expansion in powers of $1/m_b$,

$$b_v(x) = [1 + \frac{i\slashed{D}}{2m_b} + \mathcal{O}(\frac{1}{m_b^2})]h_v(x), \quad (43)$$

where $D_{\mu}$ is the covariant derivative. The effective Lagrangian is

$$\mathcal{L}_{HQET} = \bar{h}_v iv \cdot Dh_v + \bar{h}_v \frac{(iD)^2}{2m_b} h_v - \bar{h}_v \frac{gG_{\alpha\beta}\sigma^{\alpha\beta}}{4m_b} h_v + \mathcal{O}(\frac{1}{m_b^2}), \quad (44)$$

12

where $igG_{\mu\nu} = [D_\mu, D_\nu]$. The series expansion in powers of $1/m_b$ is now explicit.

By virtue of the methods based on the HQET the expansion coefficients $C_{ni}$ in Eq.(40) can be expressed in terms of small quantities, proportional to powers $\Lambda_{QCD}/m_b$; to be precise, the order of $C_{ni}$ and hence that of the moment $M_n(m_b/M_B)$ is expected to be $(\Lambda_{QCD}/m_b)^n$. Hence nonperturbative effects can, in principle, be calculated in a systematic manner. A few coefficients are calculated to be

$$C_{10} = \frac{5m_b}{3M_B}E_b + \mathcal{O}(\Lambda_{QCD}^3/m_b^3), \tag{45}$$

$$C_{11} = -\frac{2m_b}{3M_B}E_b + \mathcal{O}(\Lambda_{QCD}^3/m_b^3), \tag{46}$$

$$C_{20} = \frac{2m_b^2}{3M_B^2}K_b + \mathcal{O}(\Lambda_{QCD}^3/m_b^3), \tag{47}$$

$$C_{21} = C_{22} = 0, \tag{48}$$

where the dimensionless parameters, which parametrize the nonperturbative phenomena, are defined as [7]

$$K_b \equiv -\frac{1}{2M_B}\langle B \left| \bar{h}_v \frac{(iD)^2}{2m_b^2} h_v \right| B \rangle, \tag{49}$$

$$G_b \equiv \frac{1}{2M_B}\langle B \left| \bar{h}_v \frac{gG_{\alpha\beta}\sigma^{\alpha\beta}}{4m_b^2} h_v \right| B \rangle, \tag{50}$$

with $E_b = K_b + G_b$. Both parameters are expected to be order $(\Lambda_{QCD}/m_b)^2$.

According to Eqs.(45-48) and (42), the first two moments of the light-cone distribution function are

$$M_0(m_b/M_B) = 1, \tag{51}$$

$$M_1(m_b/M_B) = \frac{m_b}{M_B}E_b + \mathcal{O}(\Lambda_{QCD}^3/m_b^3), \tag{52}$$

$$M_2(m_b/M_B) = \frac{2m_b^2}{3M_B^2}K_b + \mathcal{O}(\Lambda_{QCD}^3/m_b^3). \tag{53}$$

$M_0$ is exactly equal to 1. $M_1(m_b/M_B)$ receives no contribution of order $\Lambda_{QCD}/m_b$. As a consequence, there are no nonperturbative QCD corrections to moments at the level $\Lambda_{QCD}/m_b$; they arise first at order $(\Lambda_{QCD}/m_b)^2$.

Substituting Eqs.(52) and (53) into Eqs.(36) and (37), the mean $\mu$ and the variance $\sigma^2$ of the light-cone distribution function $f(\xi)$ are estimated up to $(\Lambda_{QCD}/m_b)^2$ corrections to be

$$\mu = \frac{m_b}{M_B}(1 + E_b), \tag{54}$$

$$\sigma^2 = (\frac{m_b}{M_B})^2(\frac{2K_b}{3} - E_b^2). \tag{55}$$

Therefore, the light-cone distribution function $f(\xi)$ is sharply peaked around $\xi = \mu \approx m_b/M_B$ and its width is of order $\Lambda_{QCD}/M_B$, in agreement with rather general expectations.

We can furthermore implement a numerical analysis. The parameter $G_b$ for the B meson can be related to observables [7]

$$m_b G_b = -\frac{3}{4}(M_{B^*} - M_B). \tag{56}$$

The experiment determines $G_b$ to be $-0.0065$. The parameter $K_b$ was estimated using a QCD sum rule [15]. It has a large uncertainty and its range could be $K_b = 0.006 - 0.012$. In Tables 1 and 2 we list $\mu$ and $\sigma^2$ for various parameters $m_b$ and $K_b$, respectively. The numerical evaluation indicates that $\mu = 0.85 - 0.95$ and $\sigma^2 = 0.003 - 0.007$, should the b-quark mass and the parameter $K_b$ vary within the limits $4.5\ GeV \leq m_b \leq 5.0\ GeV$ and $0.006 \leq K_b \leq 0.012$.

A few remarks are in order:

(1) The free-quark decay model is reproduced, if one keeps only the first term in the series (41), i.e., $f(\xi) = \delta(\xi - m_b/M_B)$. The nonperturbative corrections manifest themselves in the second and higher terms in Eq.(41).

(2) Important information about the light-cone distribution function is obtained by applying the OPE and the HQET. The first two moments of it are reduced to two accesible parameters $G_b$ and $K_b$. However, it should be emphasized that the first few moments do not exhaust the information hidden in the distribution function, because they do not determine it completely. This point becomes obvious when one observes that any truncated resummation of the expansion in Eq.(34) cannot produce a smooth function. This is the origin of the singularity at the upper endpoint of the electron energy spectrum found in

[6, 7, 8], where a truncated HQET-based OPE has been used. Moreover, a truncated series gives rise to a delta function in the hadronic tensor $W_{\mu\nu}$, which demands the decay to be described by quark kinematics instead of hadron kinematics. This brings about ambiguities particularly at the endpoint of the $b \to u$ electron energy spectrum. In quark kinematics the endpoint of the $b \to u$ electron energy spectrum lies at $E_e = m_b/2$, while the actual endpoint should be $E_e = M_B/2$ from kinematics at hadron level.

As a matter of fact, an infinite number of terms in the light-cone OPE (39) must be included, and we cannot reduce our task to the calculation of a few matrix elements of lower dimension operators. It should be noted that, when more moments are taken into account, then higher dimensional operators of the OPE are involved; although qualitatively the moment $M_n(m_b/M_B)$ is expected to be of order $(\Lambda_{QCD}/m_b)^n$ in the framework of the OPE and the HQET, their hadronic matrix elements are much more difficult to calculate. We may conclude that the OPE and the HQET can serve as an useful technique for obtaining additional information on nonperturbative QCD, but they alone are not sufficient to determine the shape of the heavy quark light-cone distribution function. The limitations of this method must be complemented by other theoretical approaches. Alternatively, the required information may become available from experiment. For example, the light-cone distribution function can be determined from a measurement of the triple differential decay rate $d^3\Gamma/dE_e dq^2 d\xi_+$, as advocated in [16].

(3) A resummation of the operator product expansion has been performed recently in [17, 18, 19] in order to eliminate the difficulties mentioned previously in the approach for inclusive B decays [6, 7, 8]. Their treatments using an operator product expansion in the context of the $1/m_b$ expansion is different in an essential way from what we have formulated here. The distribution function is introduced in section 4 in a general way (without invoking the HQET), in analogy to DIS. In the present work the HQET is employed to estimate moments of the light-cone distribution function. To this end one may use other nonperturbative approaches (e.g. QCD sum rules). Moreover, the physical interpretation of distribution functions is distinct: the structure function $f(k_+)$ defined in [19] determines the probability to find a $b$ quark with the light-cone residual momentum $k_+$ inside the B

meson, while the light-cone distribution function $f(\xi)$ defined by Eq.(22) is the probability of finding a $b$ quark carrying momentum $\xi P_B$ within the B meson. The differences originate drastically different predictions.

# 7   Model-independent Predictions

The light-cone distribution function contains the long-distance physics associated with strong interactions of the $b$ quark inside the B meson. Although we know some important properties of this function derived on general grounds and remarkable progress was made in calculating its first few moments, it cannot yet be determined completely from first principles. Nevertheless, it is interesting to draw model-independent results without relying on the quantitative aspect of distribution functions.

It is convenient to define the scaling structure functions $F_i$

$$F_1(\xi_+, q^2) = \frac{1}{2}W_1(\xi_+, q^2), \tag{57}$$

$$F_2(\xi_+, q^2) = \frac{\xi_+ - \xi_-}{8}W_2(\xi_+, q^2), \tag{58}$$

$$F_3(\xi_+, q^2) = \frac{\xi_+ - \xi_-}{4}W_3(\xi_+, q^2), \tag{59}$$

$$F_5(\xi_+, q^2) = \frac{\xi_+ - \xi_-}{4}W_5(\xi_+, q^2). \tag{60}$$

The kinematical analysis shows that in the light-cone domain and away from the resonance region, namely in the kinematical region where our approach applies, $f(\xi_-)$ is expected to be relatively small and can be ignored, since the light-cone distribution function is sharply peaked around $\xi = \mu \approx m_b/M_B$, as established by the HQET analysis in the last section. We anticipated this, because the function $f(\xi_-)$ describes the creation of a quark-antiquark pair inside the B meson through Z-diagram and the virtual correction should be small. Then Eqs.(24-28) are simplified to

$$\xi_+ F_1(\xi_+, q^2) = F_2(\xi_+, q^2) = -\xi_+ F_3(\xi_+, q^2) = -\xi_+ F_5(\xi_+, q^2) = \xi_+ f(\xi_+). \tag{61}$$

Two important features of these expressions are:

(i) the structure functions $F_i$ satisfy scaling: they become functions of $\xi_+ = [q \cdot P_B +$

$\sqrt{(q \cdot P_B)^2 - M_B^2(q^2 - m_q^2)]}/M_B^2$ alone and are independent of the momentum transfer squared $q^2$;

(ii) the structure functions are related to each other through the light-cone distribution function.

Thus the structure functions $F_i(\xi_+, q^2)$ are measures of the momentum distribution of the b quark in the decaying B meson. The first result is the analogue of the Bjorken scaling in B-decays. The second one will be evidence for the spin-1/2 nature of charged partons (the quarks), i.e. the analogue of the Callan-Gross relation. Furthermore, using Eq.(61) and the normalization of the light-cone distribution function leads to the following sum rules:

$$\int_0^1 d\xi_+ F_1(\xi_+, q^2) = \int_0^1 \frac{d\xi_+}{\xi_+} F_2(\xi_+, q^2) = -\int_0^1 d\xi_+ F_3(\xi_+, q^2)$$
$$= -\int_0^1 d\xi_+ F_5(\xi_+, q^2) = \int_0^1 d\xi_+ f(\xi_+) = 1. \tag{62}$$

These results are very similar to those in DIS, since the behavior of the hadronic tensors for both cases is dictated by the light-cone dynamics. The remarkable thing is that these results follow in general without having information about the specific shape of the light-cone distribution function. It is important to keep in mind that the above results are valid up to perturbative and non-leading nonperturbative QCD corrections. As in DIS including the perturbative QCD corrections will lead to scaling violation: quantities which scale will be modified by powers of $lnq^2$. In order to uncover these properties a detailed measurement of the differential decay rate $d^3\Gamma/dE_l dq^2 dq_0$, Eq.(9), and hence of the structure functions $F_i(\xi_+, q^2)$ is essential.

# 8　Electron Energy Spectra

The results obtained so far take into account the lepton mass effects. Now we concentrate on the inclusive semileptonic B decay to electrons, in which the electron mass is negligible. The differential decay rates for $\bar{B} \to e\bar{\nu}_e X_q$ in the B rest frame is then simplified from

Eq.(30) to be

$$\frac{d^3\Gamma}{dE_e dq^2 dq_0} = \frac{G_F^2 |V_{qb}|^2}{4\pi^3} \frac{q_0 - E_e}{\sqrt{\mathbf{q^2} + m_q^2}} \{f(\xi_+)(2E_e\xi_+ - q^2/M_B) - (\xi_+ \leftrightarrow \xi_-)\}, \qquad (63)$$

where the variables $\xi_\pm$ given in (29) become

$$\xi_\pm = \frac{q_0 \pm \sqrt{\mathbf{q^2} + m_q^2}}{M_B} \qquad (64)$$

with $q = c, u$ relevant to the $b \to c$ and the $b \to u$ decays, respectively.

As already pointed out, appropriately including nonperturbative QCD effects allows us to use the actual kinematical limits at hadron level and to give the correct $E_e$ end point. They are given by:

$$0 \leq \ E_e \ \leq \frac{M_B}{2}(1 - \frac{M_{X_{min}}^2}{M_B^2}), \qquad (65)$$

$$0 \leq \ q^2 \ \leq 2E_e(M_B - \frac{M_{X_{min}}^2}{M_B - 2E_e}), \qquad (66)$$

$$E_e + \frac{q^2}{4E_e} \leq \ q_0 \ \leq \frac{q^2 + M_B^2 - M_{X_{min}}^2}{2M_B}. \qquad (67)$$

In Figs. 1 and 2 the phase spaces for the $b \to c$ and the $b \to u$ decays are demonstrated, respectively, which show also domains of validity for our approach. In the resonance region bound-state effects in the final state become large. However, physical quantities, integrated over an appropriate phase space region, could be calculated reliably in our approach.

In order to calculate decay distributions, one needs an Ansatz for the light-cone distribution function $f(\xi)$ consistent with the general properties pointed out in section 5. We propose a parametrization for the light-cone distribution function with two parameters $a$ and $b$ as follows

$$f(\xi) = N\frac{\xi(1 - \xi)}{(\xi - b)^2 + a^2} , \qquad (68)$$

where N is the normalization constant. In addition, constraints on the parameters $a$ and $b$ are imposed by the numerical evaluation of the mean value and the variance of the light-cone distribution function implemented in section 6 based on the techniques of the OPE and the HQET. Within the bounds of Tables 1 and 2, we find $a = 0.002 - 0.016$ and $b = 0.86 - 0.97$. Our light-cone distribution function is illustrated in Fig. 3.

Next, we use the distribution function (68) to compute the electron energy spectra for both $b \to c$ and $b \to u$ decays, shown in Figs. 4 and 5 respectively. We see that both spectra are smooth and go to zero at the endpoint. There is no sharp spike as $E_e$ approaches the upper endpoint. A desirable consequence is that perturbative QCD corrections on top of our spectra will give finite results, i.e. without endpoint singularities, because of the vanishing of the spectra at the endpoint. Therefore the endpoint behaviour of the spectra with perturbative QCD corrections are smooth for both $b \to u$ as well as $b \to c$ decays.

Comparing with the free-quark decay model, we find that nonperturbative QCD corrections appear to be significant for the spectra in the endpoint region, referring to Figs. 4 and 5. In particular, nonperturbative QCD effects become more important for the charmless $\bar{B} \to e\bar{\nu}_e X_u$ decay spectrum in the endpoint region. Our $b \to u$ endpoint spectrum is considerably softer than the free-quark decay spectrum, which is finite and non-zero at the endpoint.

# 9   Conclusions

We studied the semileptonic and inclusive B meson decays using field theoretic methods, which justifies several steps of the parton model. We give in terms of a light-cone distribution function the general formulas for the $\bar{B} \to l\bar{\nu}_l X_q$ decays, keeping the mass of the final quark $m_q = m_u$ or $m_c$. These formulas should be useful for analysing the decay spectra and determining the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$.

Additional properties of the distribution function are derived from the HQET. These are the first two moments. They are useful but do not determine the distribution function completely. The distribution function must be determined, presently, from detailed fits of the data. For this reason we discuss a new parametrization of the distribution function, which in the heavy quark limit reduces to a delta function and thus reproduces the free-quark decay model.

There is now at our disposal a complete and reliable formalism for inclusive semileptonic B meson decays. It can be used directly to fit experimental data. This could be

19

done, preferably, by experiment groups, which can appropriately include their experimental conditions. We are also working along the line trying to reproduce the electron energy spectrum and other features of the data. Early comparisons of the parton-model spectrum with experimental data are encouraging [3].

The approach is more reliable for the charmless $\bar{B} \to e\bar{\nu}_e X_u$ decays and shows that a large percentage of the events (more than 15%) has $E_e > 2.3$ GeV, which may result only from the $b \to u$ transition. It should be possible to analyse the endpoint spectrum in our approach including radiative corrections and obtain a reliable value for $V_{ub}$.

**Acknowledgements**

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Table 1 The mean $\mu$ of the light-cone distribution function for several values of $m_b$ and $K_b$.

| $m_b[GeV]$ | 4.5 | 4.6 | 4.7 | 4.8 | 4.9 | 5.0 |
|---|---|---|---|---|---|---|
| $K_b = 0.006$ | 0.849 | 0.867 | 0.886 | 0.905 | 0.924 | 0.943 |
| $K_b = 0.007$ | 0.849 | 0.868 | 0.887 | 0.906 | 0.925 | 0.944 |
| $K_b = 0.008$ | 0.850 | 0.869 | 0.888 | 0.907 | 0.926 | 0.945 |
| $K_b = 0.009$ | 0.851 | 0.870 | 0.889 | 0.908 | 0.927 | 0.946 |
| $K_b = 0.010$ | 0.852 | 0.871 | 0.890 | 0.909 | 0.928 | 0.947 |
| $K_b = 0.011$ | 0.853 | 0.872 | 0.891 | 0.910 | 0.929 | 0.948 |
| $K_b = 0.012$ | 0.854 | 0.873 | 0.892 | 0.911 | 0.930 | 0.948 |

Table 2 The variance $\sigma^2$ of the light-cone distribution function for several values of $m_b$ and $K_b$.

| $m_b[GeV]$ | 4.5 | 4.6 | 4.7 | 4.8 | 4.9 | 5.0 |
|---|---|---|---|---|---|---|
| $K_b = 0.006$ | 0.00288 | 0.00301 | 0.00314 | 0.00328 | 0.00342 | 0.00356 |
| $K_b = 0.007$ | 0.00336 | 0.00351 | 0.00367 | 0.00383 | 0.00399 | 0.00415 |
| $K_b = 0.008$ | 0.00384 | 0.00401 | 0.00419 | 0.00437 | 0.00456 | 0.00474 |
| $K_b = 0.009$ | 0.00432 | 0.00451 | 0.00471 | 0.00492 | 0.00512 | 0.00533 |
| $K_b = 0.010$ | 0.00480 | 0.00501 | 0.00523 | 0.00546 | 0.00569 | 0.00592 |
| $K_b = 0.011$ | 0.00527 | 0.00551 | 0.00575 | 0.00600 | 0.00625 | 0.00651 |
| $K_b = 0.012$ | 0.00574 | 0.00600 | 0.00627 | 0.00654 | 0.00681 | 0.00709 |

**Figure Captions**

1. The phase space for the $b \rightarrow c$ inclusive semileptonic decay of B-mesons. The dashed and dotted lines encircles the domain of validity for our approach. The region between the solid and the dashed curves is the resonance region.

2. Same as Fig.1, but for the $b \rightarrow u$ decay.

3. The light-cone distribution function (68). The parameters are taken to be $a = 0.0076$ and $b = 0.92$.

4. The electron energy spectrum in $\bar{B} \rightarrow e\bar{\nu}_e X_c$ decays. The solid line is obtained in our approach for $M_{X_{min}} = m_c = 1.5\ GeV$, $a = 0.0076$ and $b = 0.92$. The dashed line corresponds to the free-quark decay model spectrum, using $m_b = 5.0\ GeV$ and $m_c = 1.7\ GeV$ set by a fit to ARGUS data.

5. Same as Fig.4, but for the $b \rightarrow u$ decay. The solid line is predicted in our approach for $M_{X_{min}} = m_u = 0$, $a = 0.0076$ and $b = 0.92$. The dashed line results from the free-quark decay model, using $m_b = 5.0\ GeV$ and $m_u = 0$.
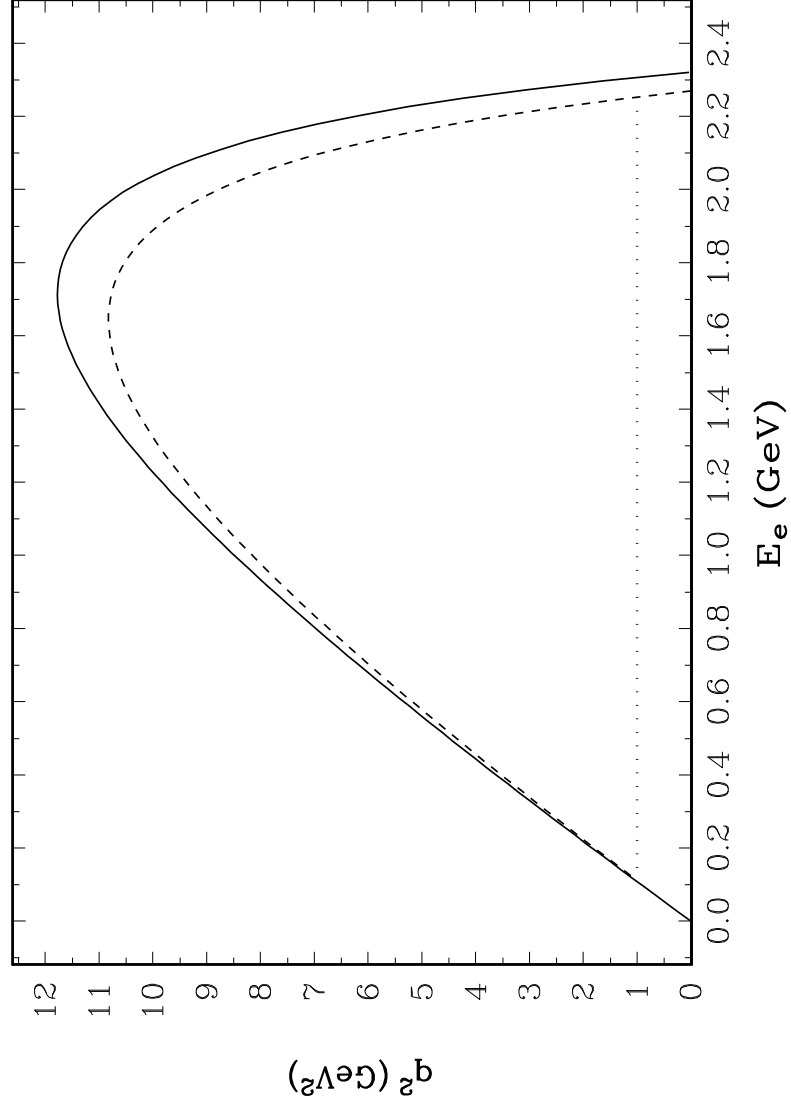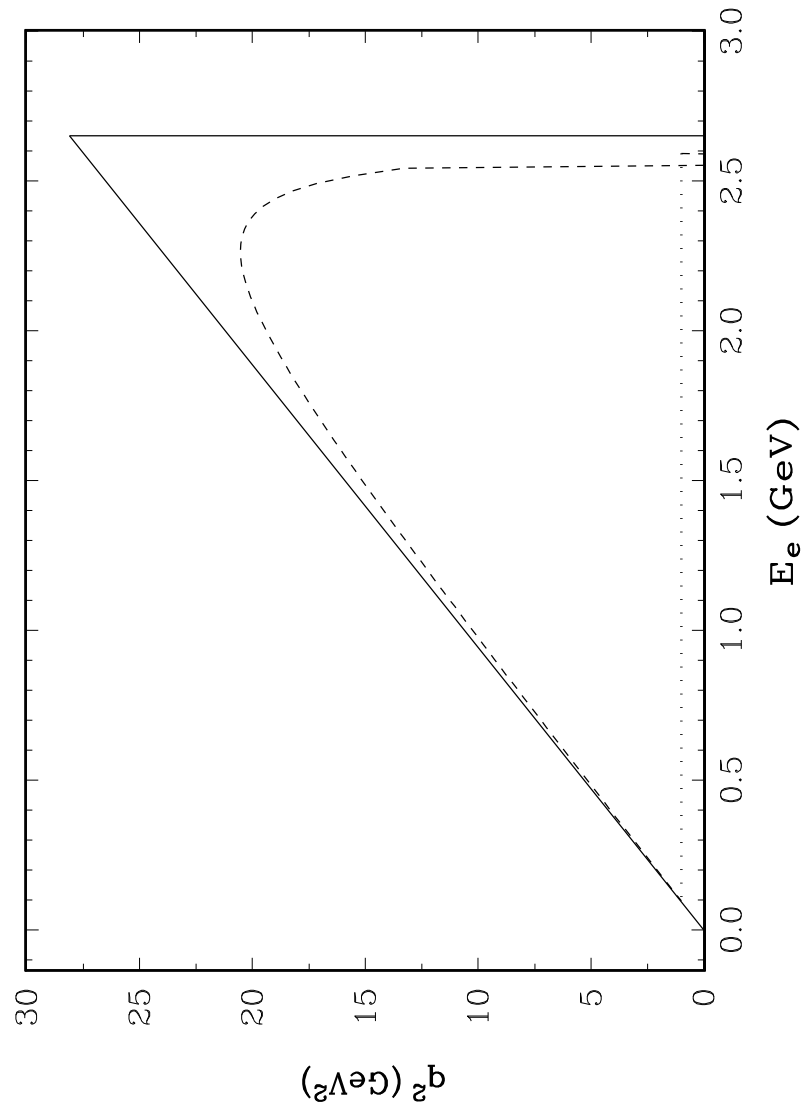
Fig. 1

Fig. 2

Fig. 3

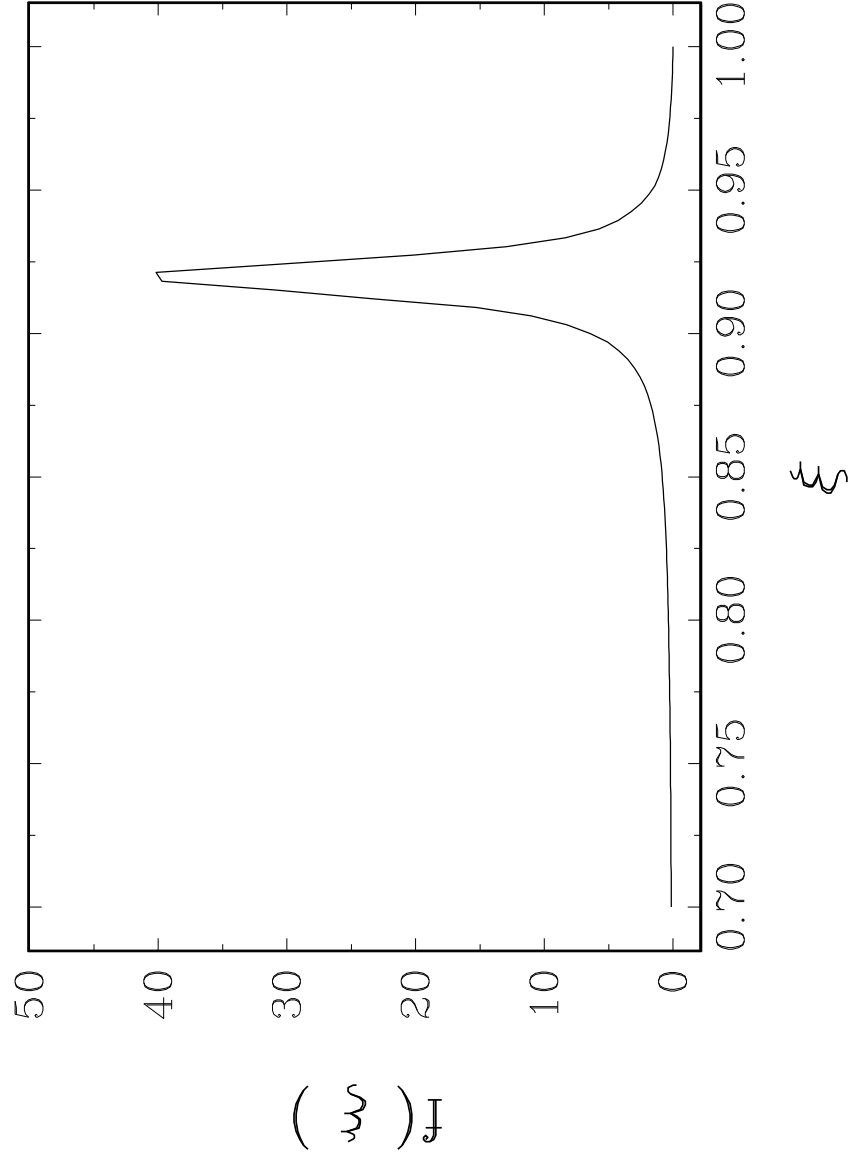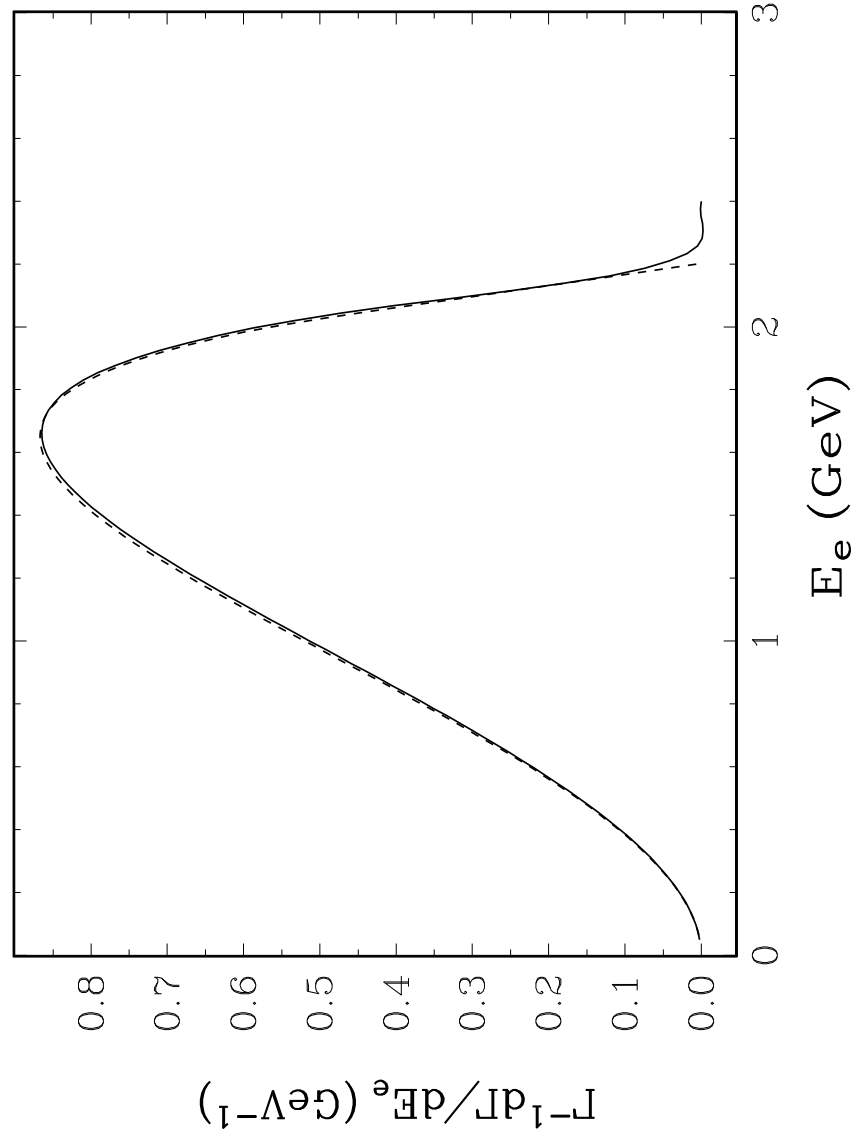Fig. 4

Fig.5